

## Problem Formulation

**Model:** A Stochastic Cyber-Physical System  $\Sigma$  whose output is modeled as a parameterized stochastic process  $Y(t, \theta)$ . The parameter  $\theta$  is a couple  $(x_0, u)$ , where  $x_0$  is the initial condition of the system, and  $u$  parameterizes the input signal to  $\Sigma$ . The randomness can be the result of sensor noise and other physical factors. All testing happens within a bounded time domain  $R$ .

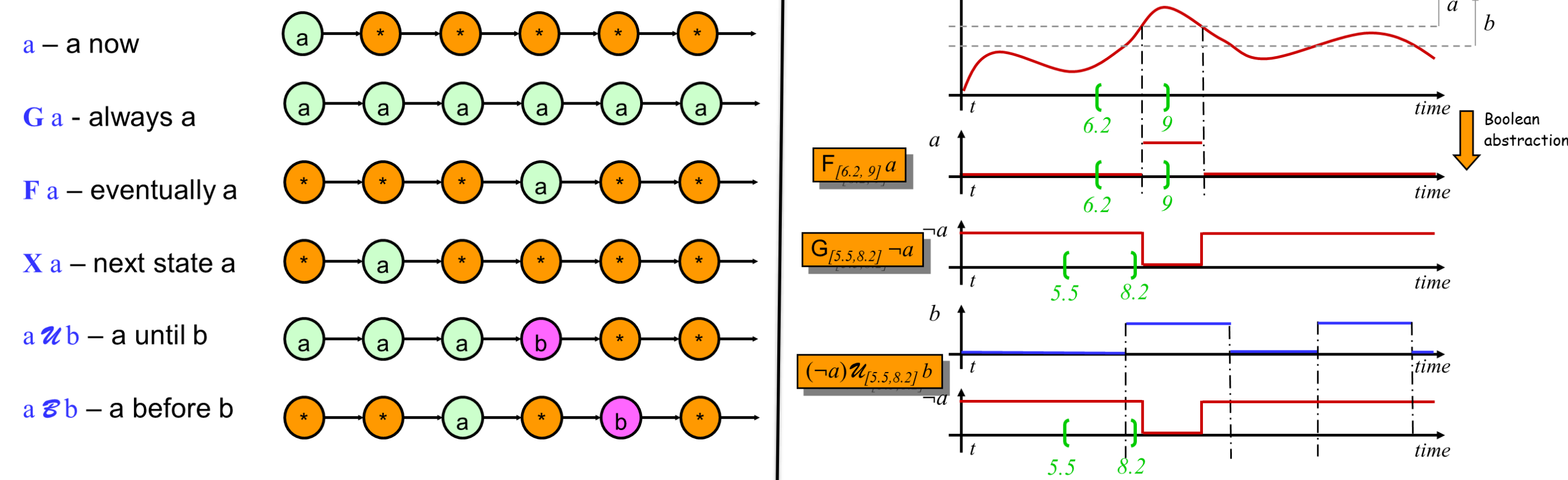
**Specification:** A metric temporal logic (MTL) formula  $\phi$  that captures the system's desired behavior.

**Problem:** For an MTL specification  $\phi$ , the falsification problem for SCPS consists of finding a parameter value  $\theta = (x_0, u)$  of the system  $\Sigma$  such that, *on average*,  $\Sigma$  driven by  $\theta$  does not satisfy specification  $\phi$ .

## Average MTL robustness

The formulas are built from a finite number of atomic propositions which label regions of interest in the state space. The propositional formulas are formed using the traditional operators of *conjunction* ( $\wedge$ ), *disjunction* ( $\vee$ ), *negation* ( $\neg$ ), *implication* ( $\Rightarrow$ ), and *equivalence* ( $\Leftrightarrow$ ). MTL formulas are obtained from the standard propositional logic by adding temporal operators such as *eventually* ( $\Diamond$ ), *always* ( $\Box$ ), and *until* ( $U$ ). MTL also allows timing constraints.

### MTL intuition



### Robustness of MTL formulae

**Robustness** is a functional that associates a real extended number to each sample path  $y$  of  $Y$ . A positive robustness indicates that the signal satisfies the formula, and a negative value indicates that it falsifies it.

$$\rho_\phi : y(\cdot, \omega; \theta) \mapsto \rho_\phi(y(\cdot, \omega; \theta)) \equiv \rho_\phi(\omega, \theta) \in [-\infty, \infty]$$

The *average robustness*  $U(\theta)$  captures the average behavior of the system for that  $\theta$ . We minimize  $U(\theta)$  to find worst-case average behavior.

$$U(\theta) = \mathbb{E}_P[\rho_\phi(\omega, \theta)] = \int_{\Omega} \rho_\phi(\omega, \theta) dP(\omega)$$

## Robustness minimization and Statistical MC

Specification: the normalized air-to-fuel ratio is always within [0.9, 1.1]



Worst-case average ratio = 1.2

Worst-case average ratio = 2.1

P. Zuliani, A. Platzer, and E. M. Clarke. Bayesian statistical model checking with application to simulink/stateflow verification, HSCC 2010

## Robustness-Guided Temporal Logic Testing for Stochastic Hybrid Systems

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Sponsors: CNS 1116136, CNS 1319560, IIP-0856090

### Recent examples of Automotive Recalls due to CPS Errors (2011-2012)

No downshifting from 5<sup>th</sup> to 4<sup>th</sup> under certain operating conditions

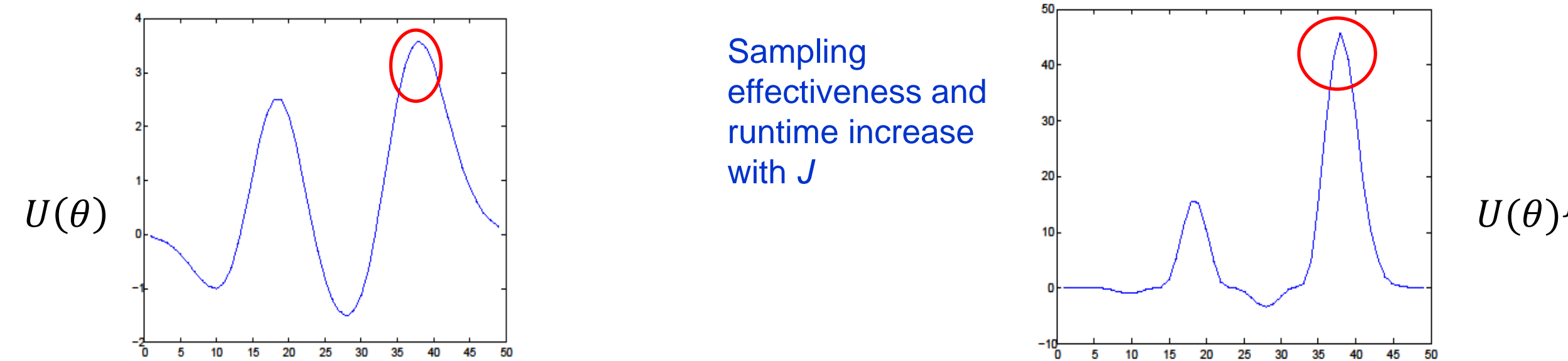
Rough idling or stalling due to complicated adaptive ECU

Cruise control does not disengage unless turning off the ignition

## Minimization and guarantees

$$U_* = \inf \{U(\theta) | \theta \in \Theta\}$$

Use a variant of Simulated Annealing adapted to minimizing expectations.

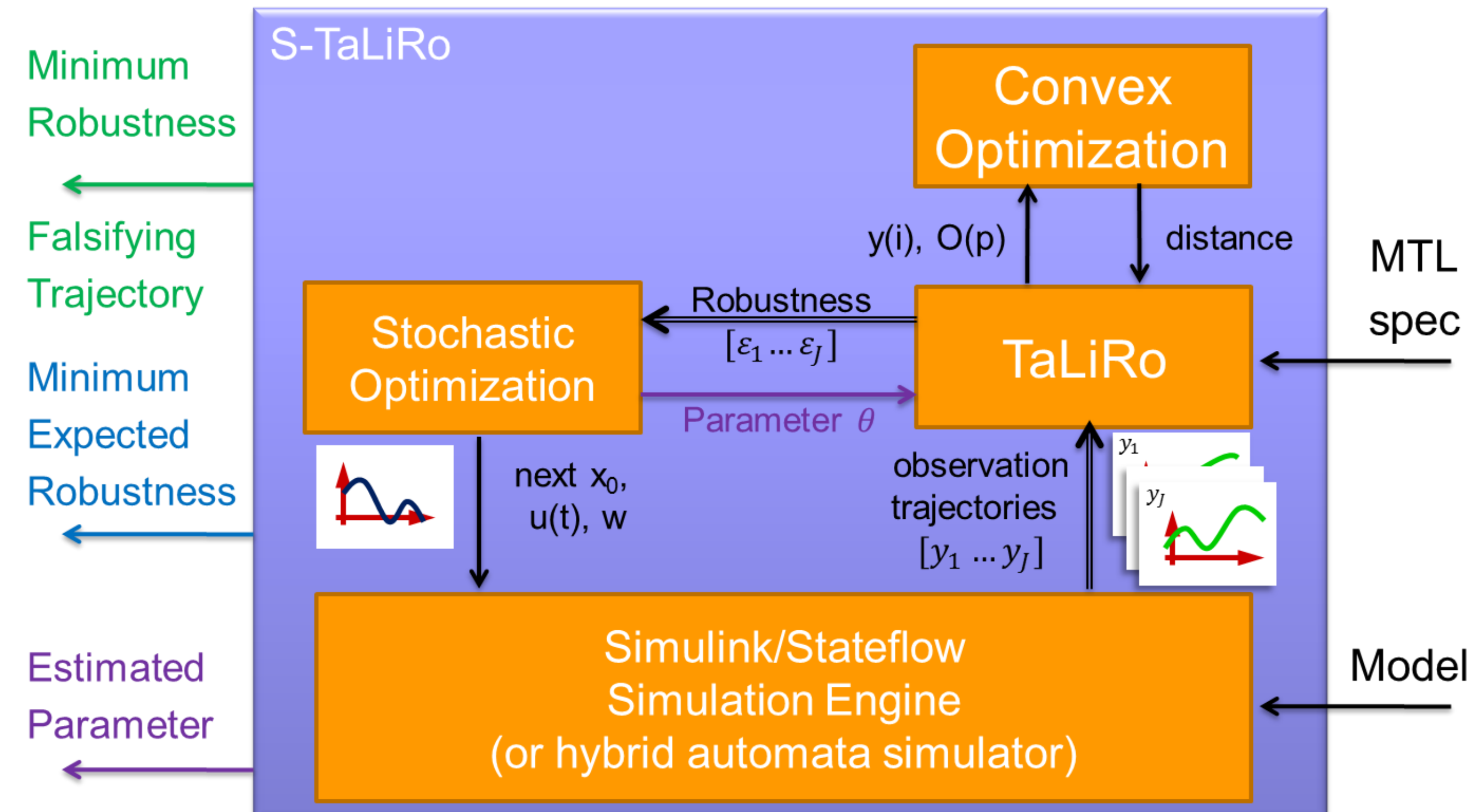


Example guarantee: for a given  $\varepsilon > 0$  and  $\delta > 0$ , find number of samples s.t.

$$\Pr[|U(\theta_\varepsilon) - U(\theta_*)| < \varepsilon] > \delta$$

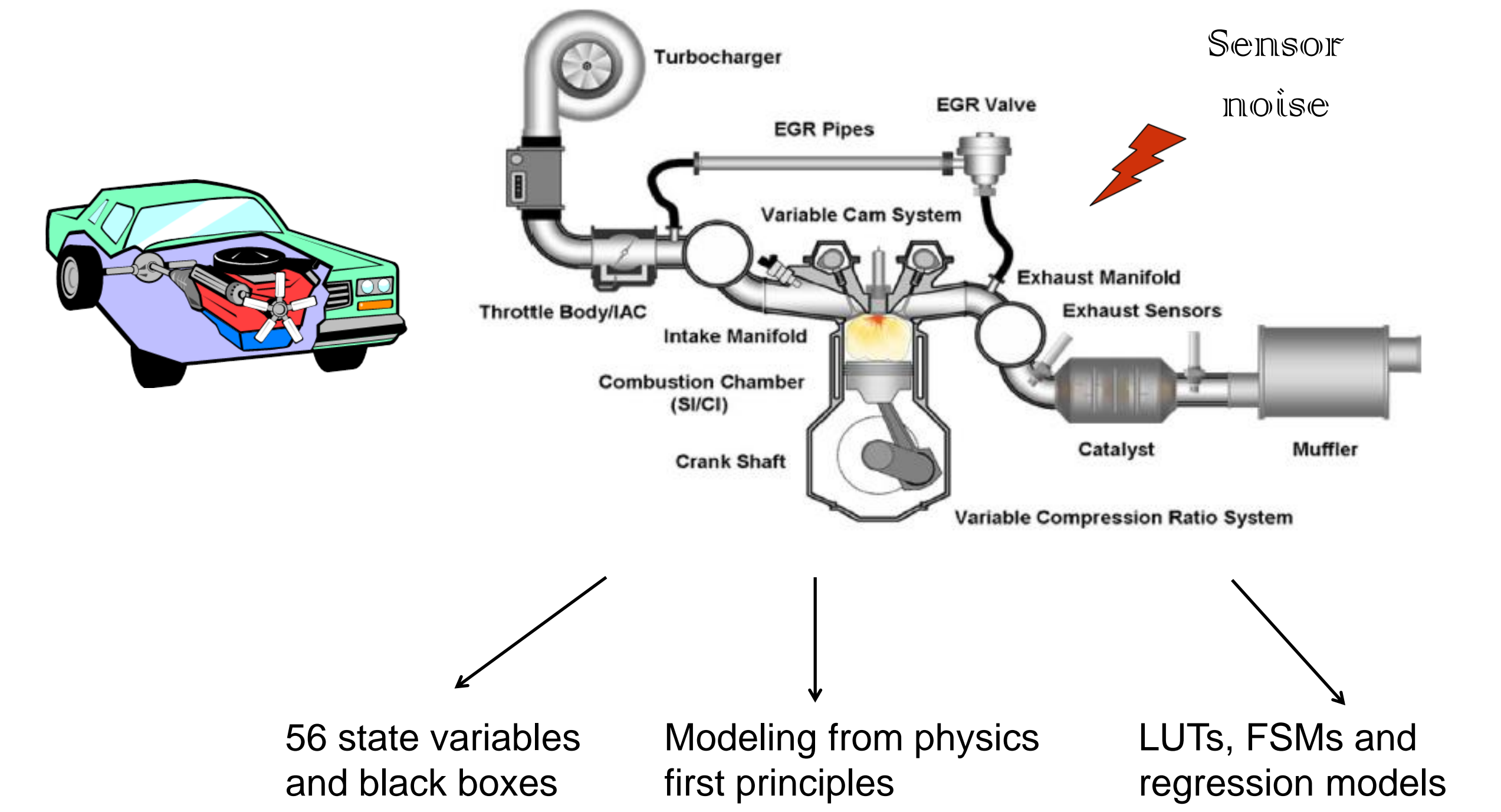
A. Lecchini, J. Lygeros, and J. M. Maciejowski. Stochastic optimization on continuous domains with finite-time guarantees by Markov chain Monte Carlo methods. IEEE Transactions on Automatic Control, 2010.

## Implemented in the S-TaLiRo Toolbox



- Finding Falsifying Trajectories for Deterministic Cyber Physical Systems
  - Minimum Expected Robustness for Stochastic Cyber Physical Systems
  - Parameter Estimation of MTL formulas for Cyber Physical Systems
- [www.tinyurl.com/Staliro](http://www.tinyurl.com/Staliro)

## High-fidelity engine model with sensor noise



SimuQuest® engine model

Find values for the parameter vector  $\theta$  such that  $\phi$  is falsified:

Formal Specification  $\phi$

Whenever the normalized air-to-fuel ratio is outside [0.9, 1.1], it will settle back inside the range within 1 sec, and stay there for at least 1 sec.

$$\phi = G_{[0,102]}(\text{OutOfBounds} \rightarrow F_{[0,1]}(G_{[0,1]} \text{InBounds}))$$

## Verification results

